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A Unified Interpretation of Room-Temperature Strength of Notched Specimens as Influenced by Their Size

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This paper proposes a generalized concept of material behavior in the presence of sharp notches or cracks, which embraces the notch-strengthening of small bars and the notch-weakening of large bars. Quantitative explanation of notch-strengthening of small bars follows from the experimental work of G. Sachs and others, and the interpretation of notch-weakening of large bars, the geometric size effect, is obtainable with the help of Griffith-Irwin approach using the strain-energy-release concept. Experimental notch strength versus size curves for notch-tension and notch-bending tests are plotted in an especially useful log-log co-ordinate system. An analytical expression was derived to approximate the foregoing experimentally observed, continuous variation of notch strength. A relationship was derived between the notch strength and relative notch depth for cylindrical tension bars with different outside diameters. The plots indicate a continuity of notch-strength behavior with varying size and varying notch depth.

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Nomenclature

The following nomenclature is used in this paper:

- c = depth of notch, in.
- d = notch diameter, also gross depth of beam, in.
- D = outside diameter of bar, in.
- E = modulus of elasticity, psi
- FATT = Charpy-V-notch fracture appearance (50 per cent fibrous) transition temperature, deg F
- G = strain-energy release rate, in-lb per sq in.
- G_c = fracture toughness, in-lb per sq in.
- G_{co} = fracture toughness in the presence of crack, in-lb per sq in.
- h = net depth of beam at notch, in. = $d - c$
- M = bending moment, lb-in.
- N = notch depth, ratio of area removed by the notch to gross cylindrical area = $1 - (d/D)^2$
- N_t = notch-strength ratio in tension = S_d/S_t
- P = axial force, lb
- P_m = maximum axial force in a tension test, lb
- p = coordinate difference = $x_2 - x_1$
- q = coordinate difference = $y_2 - y_1$
- r = radius at the apex of notch, in.
- S = stress, psi
- S_d = notch-strength in tension test, psi = $P_m/(\pi d^2/4)$
- S_h = nominal bending stress at the notched beam section of net depth h, psi = $M/(th^2/6)$
- S_n = notch-strength, psi
- S_t = tensile strength, psi
- S_y = yield strength, psi
- t = thickness of beam, in.
- x, y = co-ordinates
- ν = Poisson's ratio

Introduction

In (1),[2] the authors have introduced the Griffith-Irwin concept for the determination of bursting speed of large, effectively notched disks. A new physical quantity, "fracture toughness," G_{c1} was experimentally determined from bursting tests of such disks made from different materials. The authors recognized that the material behavior in notched disks may be classified by using the over-all stress field near the notch apex, at the bursting speed, and comparing it with the tensile strength and yield strength of the material. This reasoning resulted in a somewhat arbitrary classification of material behavior into ductile, quasi-ductile, quasi-brittle, and brittle. Since then it was recognized that the foregoing classification is useful and may easily be adapted to apply to other configurations, like tension tests on sharply notched bars and on notched beams. In addition, a consistent pattern of notch-strength behavior emerged in which the linear size of sharply notched specimens, together with fracture toughness G_c became the defining parameters. In this paper the experimentally obtained "notch-strength-versus-size" curves are presented for tension tests of notched cylindrical specimens and for bending tests of notched beams. A family of such curves is plotted in an especially suitable log-log co-ordinate system and the adopted classification of material behavior is indicated in the diagram. Using the Griffith-Irwin concept, informative curves were plotted representing the notch-tension strength of cylindrical specimens as a function of relative notch depth.

Small Notched Tension Specimens Tested at Room Temperature:

There exists considerable literature on the

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[1] When this paper was written the author included information and expressed opinions believed to be correct and reliable. Because of the constant advance of technical knowledge, the widely differing conditions of possible specific application, and the possibility of misapplication, any application of the contents of this paper must be at the sole discretion and responsibility of the user.

[2] Underlined numbers in parentheses designate References at the end of the paper.

A-1

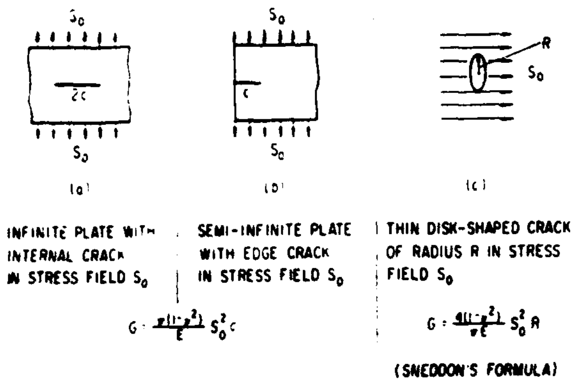

Fig.1 Some formulae for strain-energy release rate.

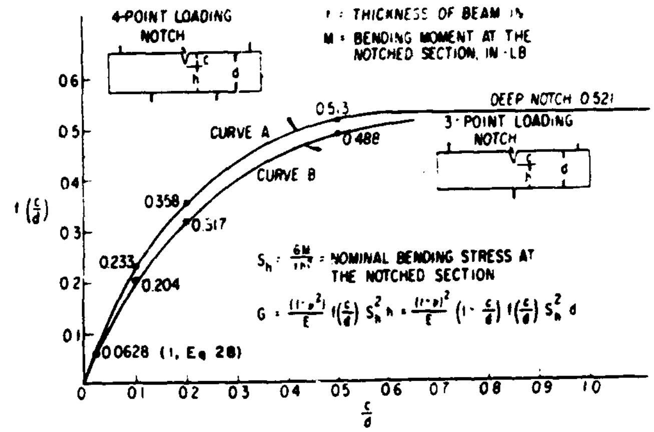

Fig.2 Strain-energy release rate for notched beams.

subject of strength of relatively small notched cylindrical tension specimens. Kunze in Germany, McAdam, Sachs, Ripling, Baldwin and their co-workers in the United States, have contributed to a better understanding of the effect of geometry on the tension behavior of cylindrical, notched specimens. Readers who intend to study the foregoing will find references (15, 16, 17, 18, 19, 20, 24) useful.

Sachs and his co-workers performed tests on heat-treated AISI 3140, 4340 and recently also on other low-alloy steels. Originally, cylindrical diameters varied between 0.300 and 0.500 in.; in later experiments diameters were increased to 3 in. Often, when notched bars are tested in tension, after the maximum has been reached the load P decreases until the bar fractures. Such behavior under notched conditions indicates that the material exhibits a degree of ductility (20, Fig.2). It has been shown that under such conditions, the relationship between the notch-strength S_d and notch depth N is approximately linear (15, p. 521); namely, it appears that for 50 per cent notches, the notch-strength S_d is about 50 per cent higher than the unnotched tensile strength S_t, and for very deep notches, almost 100 per cent deep, S_d is approaching twice S_t. This is shown in Fig.11(a) by the straight line ogm. Note that the foregoing applies only when the notch is made very sharp and when the notch angle is equal to or less than 45 or 60 deg. With increase in notch radius the notch-strength S_d passes through a maximum and then decreases as indicated in Fig.11(a), curve ogn, and in (15, Fig.5). For still larger radii, and for very deep notches, notch-strength S_d may even approach S_t, the tensile strength.

In case of steels with higher strength level, above 150,000 psi and up to 290,000 psi, notch strength deviates more and more from the straight-line relationship. If combined with very small radii, the resulting notch-strength ratio may decrease to below 1, and especially so when the notch depth is 50 to 60 per cent. It appears that for the investigated range of strength the effects of notch sharpness and stress level are interchangeable (15, Fig. 5, 7, 10) and (20, Fig.6).

Increased specimen size results in a more or less gradual deviation of the notch-strength from the straight-line relationship, especially for the high-tensile-strength steels (17, Fig.8). The combined effect of notch angle and notch depth for sharp notches is presented in a discussion by McAdam (12, Figs.16, 18). He indicates that the straight-line relationship holds well for angles less than 60 deg, for larger angles a deviation is observed.

Griffith-Irwin Quantitative Approach to Geometric Size Effect

Application of Log-Log Co-ordinates. The strengthening of notched tension, bars previously described is, as a rule, confined to bars with small diameters. With increasing diameter the notch strength decreases and may become even lower than the yield strength S_y. This notch weakening is usually accompanied by complete separation of surfaces and by fast crack propagation. Often, the main fracture surfaces have a smooth and shiny appearance indicative of cleavage fracturing and the fracture appearance is termed commonly as "brittle" (see bibliography 4, 5, 6, 7, 35, 36, 37, 38, 39 in reference 1).

The lower notch strength of larger bars may be qualitatively defined with the help of the Griffith-Irwin approach (1, page 1647). Newer references pertaining to this approach are (8, 9,

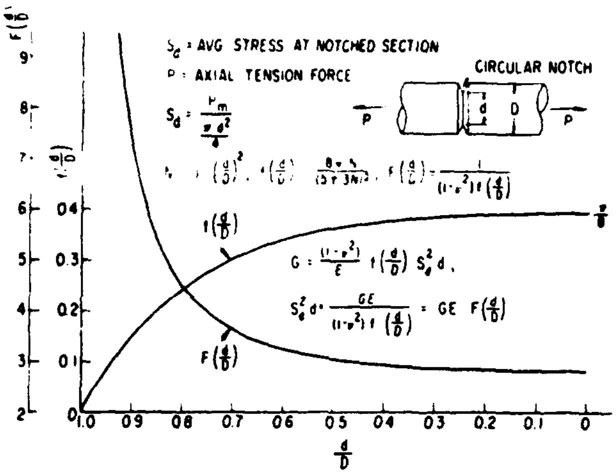

Fig.3 Strain-energy release rate for notched cylindrical bars.

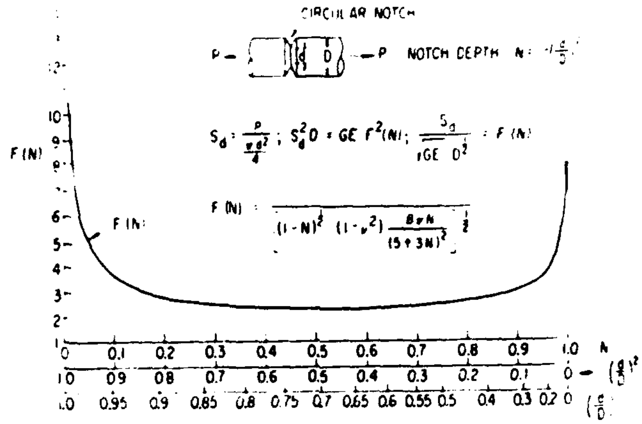

Fig.4 Notch strength of cylindrical tension specimens.

10, 11). Reference (23) is another important and very recent paper which discusses stable and unstable cracks in shear without making use of the Griffith-Irwin approach. For completeness, two definitions will be quoted from (1):

Fracture Toughness G_c. The component of work irreversibly absorbed in local plastic flow to create a unit area of fracture. Fracture toughness has also been referred to in the literature as "fracture extension force."

Strain-Energy Release Rate G. The quantity of stored elastic strain energy released from a cracking specimen as a result of extension of the advancing crack by a unit area.

Fig. 1 summarizes some of the better known expressions for strain-energy-release rate G (Fig.4 of 1). Equation (27) in (1) is an analytical expression for G, for a notched-beam specimen with 4-point loading

$$G = \left[(1 - \nu^2)/E\right] f(c/d)\ S_h^2 h \qquad (1)$$

The notched bend specimen and function (c/d) are shown in Fig. 2, curve A. Equation (1) applies to shallow, intermediate and deep notches.

The analytical expression for G for a notched beam with 3-point loading was derived in (12). The expression is the same as for the 4-point loading, but f(c/d) differs. Function f(c/d) is plotted as curve B in Fig. 2.

In order to investigate the effect of size on strength of notched beams, a series of gradually increasing notched beams is tested in slow bending. All dimensions increase proportionally and the ratio c/d is maintained constant. If we assume c/d = 0.2, i.e., if we choose a series of beams with 20 per cent notch depth and h = 0.8 d, then for 3-point loading from curve B, Fig.2

$$f(c/d) = 0.317$$

Using Equation (1),

$$S_h^2 d = 126 \times 10^6\ G \text{ psi} \qquad (2)$$

The strain-energy release rate G for a circular, sharply notched bar in tension is as follows (13):

$$G = \left[(1 - \nu^2)/E\right] f(d/D)\ S_d^2 d \qquad (3)$$

where,

$$f(d/D) = (8\pi N)/(5 + 3N)^2 \qquad (4)$$

$$S_d^2 d = GE\ F(d/D) \qquad (5)$$

$$F(d/D) = 1/\left[(1 - \nu^2) f(d/D)\right] \text{ and } S_d^2 D = GE\ F^2(N) \qquad (6)$$

where

$$F^2(N) = \left[F(d/D)\right] / (1 - N)^{1/2}$$

Functions f(d/D) and F(d/D) are plotted in Fig.3, and F(N) is plotted in Fig.4. For N = 1, i.e., for very deep notches f(d/D) = π/8. For very shallow notches d/D ≐ 1, Equation (3) practically agrees with Irwin's equation (page 6 of 27), if it is modified for plain strain.

If a particular specimen geometry is chosen, e.g., the widely used 50 per cent notch, then

$$d/D = 0.707, \quad N = 0.500$$

and from Equation (4) f(d/D) = 0.297. Therefore,

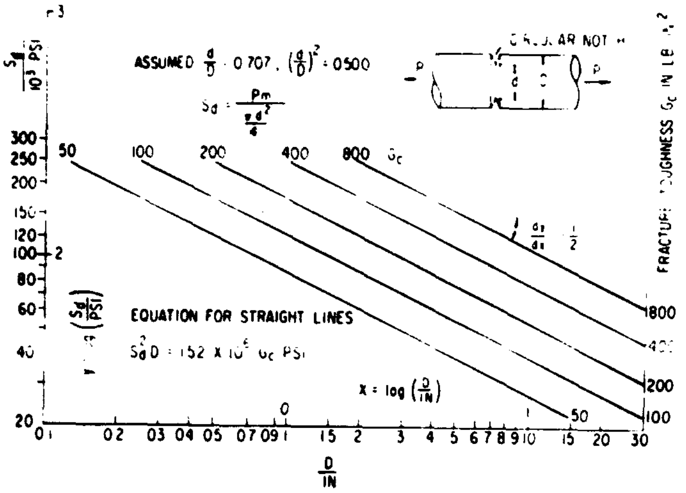

Fig.5 Notch strength of 50 per cent notched tension specimens.

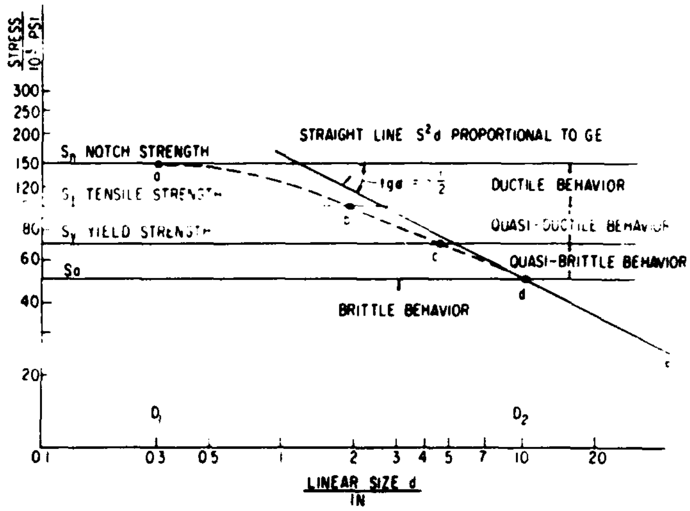

Fig.6 Notch strength versus size. Schematic representation.

$$G = 0.297\ [(1 - \nu^2)/E]\ S_d^2 d \qquad (7)$$

and, because d = 0.707 D, $\nu = 0.3$, and E = 29 x 10^6 psi

$$G = 0.0066 \times 10^{-6}\ S_d^2 D \qquad (8)$$

and

$$S_d^2 D = 152 \times 10^6\ G \text{ psi} \qquad (9)$$

It is important to notice that Equations (12), (13), (23) in (1), Equations in Fig. 1, and Equations (1), (3), and so on have the same essential form: "The product of square of stress and of a linear dimension is proportional to the strain-energy release rate." The foregoing may be interpreted as follows. "If, a series of tests is performed with proportional specimens and, if, at the instant when the crack becomes unstable and begins to propagate rapidly, all dimensions, including the crack length (or notch) are proportional, then the product of a nominal stress squared and of crack length is proportional to strain-energy-release rate G." But G at this instant becomes fracture toughness G_c, which is assumed constant and a characteristic of material. The inverse-square law which states that it takes one half of the stress to cause fracture when all dimensions increase four times, is usually designated the "geometric size effect", (1, p. 1653).

It is helpful to plot S and d in a log-log co-ordinate system with abscissa X = log d and ordinates y = log S. For an assumed value of G_c and for a particular geometry any of the Equations (2, 9) will be represented by a straight line with a slope

$$\operatorname{tg}\alpha = dy/dx = -1/2$$

A family of straight lines may be drawn, each one for a different value of G_c. Using Equation (9) a number of such straight lines were drawn in Fig.5.

CONTINUITY OF MATERIAL BEHAVIOR IN THE PRESENCE OF NOTCHES

Notch-Strength-Versus-Size Curves. We have discussed the strength behavior of small notched specimens at one end of the range of sizes and the behavior of large notched specimens at the other end of the range. It is possible to show the notch-strengthening of small specimens and the notch-weakening of large specimens on the same plot. Such a combined plot becomes especially useful, if log-log co-ordinates, are used for stress and for linear specimen size. This is depicted in Fig.6, where besides the straight line dc, also the yield strength S_y, the tensile strength S_t and the notch strength S_n are shown. It appears to be justified to assume that the notch-weakening phenomenon which becomes evident when the specimen size is increased takes place in a more or less gradual manner. Consequently, it is supposed that the notch-weakening process may be represented by the dotted line a b c d.

Point a indicates the first noticeable deviation from the line S_n and at point d the notch-strength begins to follow the straight line S^2D = const. Points b and c are intersections

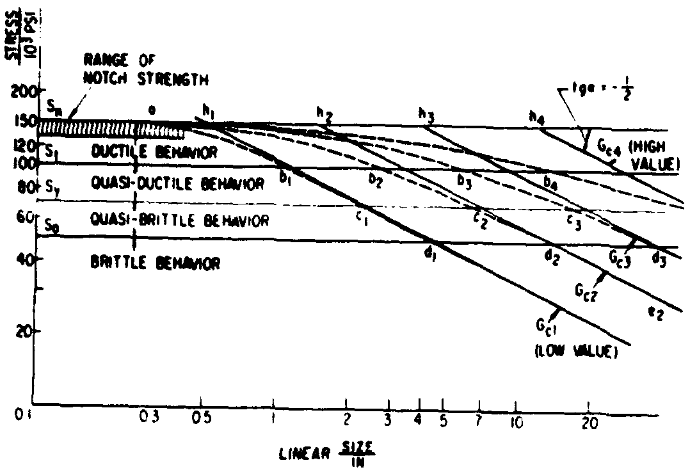

Fig.7 Family of notch strength versus size curves. Schematic representation.

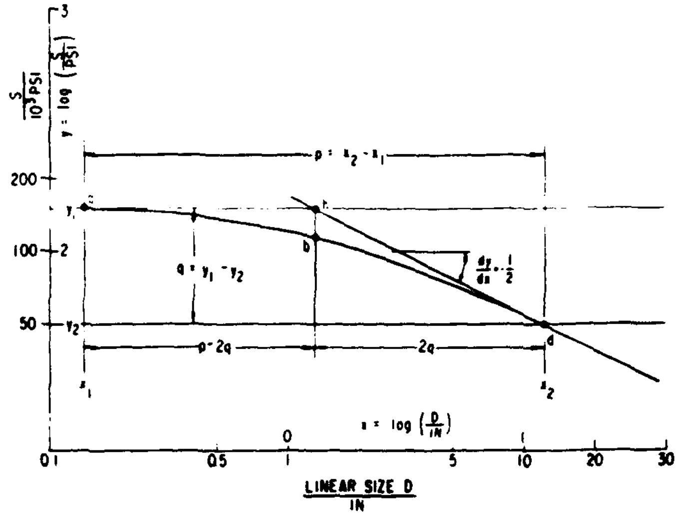

Fig.8 Notch strength versus size. Analytical representation.

with the two horizontal lines which correspond to tensile and yield strength, respectively. In this manner an assumed curve a b c d was made to depict the intermediate region of material behavior bridging the two extreme behaviors. The sizes D_1 and D_2 which define points a and d are either obtained experimentally or must be estimated. The curve "S_n a b c d e" is designated here as *"notch-strength-versus-size"* curve.

The foregoing inductive approach considerably helps to visualize the behavior of sharply notched specimens as influenced by their size. A partial confirmation of this inductive reasoning, pertaining to material behavior in the presence of notches, may be derived from two sets of experiments with notched tension and bend specimens which will be described in later sections.

Many materials, when sharply notched and when tested in small sizes at room temperature, fracture at stresses above the tensile strength S_t, as depicted by the straight line S_n in Fig.6 and by its continuation--curve a b. The small sizes are therefore notch-strengthened and the material behavior may be designated as "ductile." However, for somewhat larger sizes the notch strength decreases and falls between the tensile strength and yield strength along the curve b c. It appears logical to designate the material behavior in the region b c as "quasi-ductile." For larger sizes the notch strength falls below the yield strength S_y, as indicated by the line c d. For still larger sizes the full geometric size effect is evidenced because the notch strength follows the straight line d e.

It is therefore consistent to describe the material behavior in the region c d as "quasi-brittle" and in the region below d as "brittle."

The foregoing classification of material behavior into "ductile," "quasi-ductile," "quasi-brittle," and "brittle" is arbitrary. However, this classification appears to be consistent because the grouping is defined by means of the "over-all" stress field which occurs at the incident of fracture in the vicinity of the notch. Besides, these definitions naturally take into account the geometric size effect. An application of the foregoing may be found elsewhere (pp. 1645 and 1646 of 1). It is worthwhile to add that an attempt to classify material behavior similarly by means of fracture appearance will, for the most part, not be satisfactory. The main reason for this is that fracture appearance depends not only on the notch geometry and on the size of the specimen, but is also affected to a considerable degree by the elastic strain energy stored in the complete system which consists not only of the specimen but includes also the testing machine.

The complete "notch-strength-versus-size" curve was originated to describe test results of a series of very sharply notched but geometrically similar specimens. The relative position of straight line S_n a in respect to tensile strength S_t, and the location of line d e in respect to the abscissa, is defined only when these requirements are fulfilled. Fig.6 applies to a material heat-treated to obtain a set of physical properties and which exhibits a corresponding fracture toughness G_c. Therefore, for a particular specimen geometry, *only one notch-strength-versus-size curve* is obtainable, largely defined by the particular values of S_t and G_c. It is possible to extend the concept of the single notch-

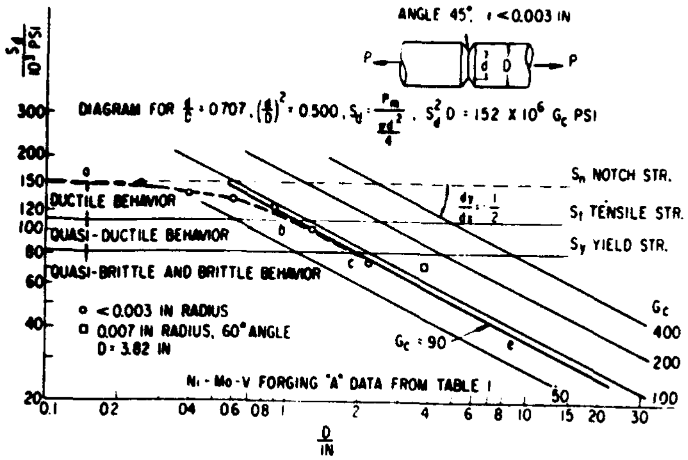

Fig.9 Notch strength versus size. Room temperature; notch-tension tests with 50 per cent notch.

strength-versus-size curve to a number of such curves forming a family. Each of such curves corresponds to a different G_c value. This is shown in Fig.7 for four different values of fracture toughness $G_{c1}, G_{c2}, G_{c3}, G_{c4}$. G_{c4} is practically 25 times as large as G_{c1}, because it is proportional to square of the stress for the same size. Of course, all four curves are for identical specimen geometry. Neither the notch-strength values nor the location of point a is definite and may be best represented by a vertically hatched area as shown in Fig.7. In drawing the curves it was assumed that the four points of tangency d_1, d_2, d_3, d_4 occur at the same stress level S_a, below the yield-strength level S_y.

The previously defined regions of material behavior are extended in Fig.7 to embrace all four curves. The cross-hatched area indicates that notch strength is hardly affected by the values of fracture toughness G_c. On the other hand, for large specimens, G_c values define the strength of sharply notched specimens and, for the same size, the notch strength of bars increases with the square root of G_c. A specimen about 0.4 in. diam has approximately 140,000 psi notch-strength, practically irrespective of fracture toughness values and behaves in a ductile manner. In case of a rather low fracture toughness G_{c1}, the material behavior enters the quasi-brittle range at C_1, when the specimen is about 2.3 in. diam. However, if the same material has been modified to result in a rather high fracture toughness G_{c3}, about 8 times G_{c1}, then the material begins to behave in a quasi-brittle manner at point C_3, at a much larger diameter, about 17 in. A very high fracture toughness G_{c4} defines curve ab_4, which indicates that for practical purposes such a material behaves in a ductile or quasi-ductile manner throughout a large range of sizes. Specimen geometry, crack depth, and size are not sufficient to define material behavior and, only when in addition fracture toughness is specified, the material behavior in terms of stress level becomes established. An important, but not very obvious conclusion may be inferred from the above discussion that theoretically, irrespective of the value of fracture toughness, sharply notched specimens will exhibit quasi-brittle and even brittle behavior if the size is increased sufficiently.

Analytical Expression for Notch-Strength-Versus-Size Curve. Having visualized the approximate shape of the notch-strength-versus-size curve, an approximate equation will be proposed for such a curve. In Fig.8 a notch-strength-versus-size curve abd is drawn, in a log-log co-ordinate system with abscissa X = lg D and ordinates y = lg S. It is prescribed that the curve abd be tangent to the line ah (notch strength) at point a and point d be tangent to the inclined line hd, whose slope is $tg\alpha = -1/2$. A parabola will fit the requirement that the curve be tangent at points a and d.

$$y_1 - y = q\left[(x - x_1)/p\right]^{p/2q} \qquad (10)$$

The power of parabola is $p/2q$, it is equal to 2 if $p = 4$, and 3 if $p = 6$. If

$$p = 4q, \quad hb = 0.25q$$
$$p = 6q, \quad hb = 0.30q$$

The author is indebted to Mr. W. G. Sweets for the derivation of Equation (10).

Room-Temperature Tests on Proportional, Sharply Notched Tension Bars

Room-temperature tests on a series of notch-tension specimens are described in (4). Test bars were taken from a heat-treated, 2.5 pct Ni - 0.55 pct Mo - 0.07 pct V forging known as material A, described in (2) and (14). The following physical properties were reported in (14):

S_y = 82,000 psi, S_t = 110,000 psi, elongation 5 to 20 per cent, RA 4 to 55 per cent, 50 per cent fibrous-fracture-appearance transition temp. (Charpy - V) - 300 deg F.

The specimens were provided with a 45-deg circumferential V-groove and had a 50 per cent notch; i.e., the ratio of d to D was maintained constant and equal to 0.707. The radius at the bottom of the notch was sharp and was less than 0.003 in. There were 12 bars tested, two of each of the following six diameters: 0.250,

TABLE 1*

TENSION TESTS OF NOTCHED BARS AT ROOM TEMPERATURE

45° Angle. Radius at Notch 0.003 in.

utside Dia. D in.	Notch Dia. d in.	Notch Area %	Max. Load P_m lb	Average Stress Across Notch S_c psi	Avg. of Tests S_d psi
250	.1775	50	3750	151,000	150,000
252	.1781	50	3720	149,000	
402	.2838	50	9000	142,000	140,500
3985	.2852	49	8880	139,000	
599	.4193	51	18100	131,000	133,500
598	.4203	51	18800	136,000	
900	.6360	50	40400	127,000	125,500
899	.6339	50	39000	124,000	
3005	.9200	50	71400	108,000	100,000
300	.911	51	59800	92,000	
250	1.593	50	146000	73,000	73,500
250	1.600	49	148000	74,000	

*From reference (4).

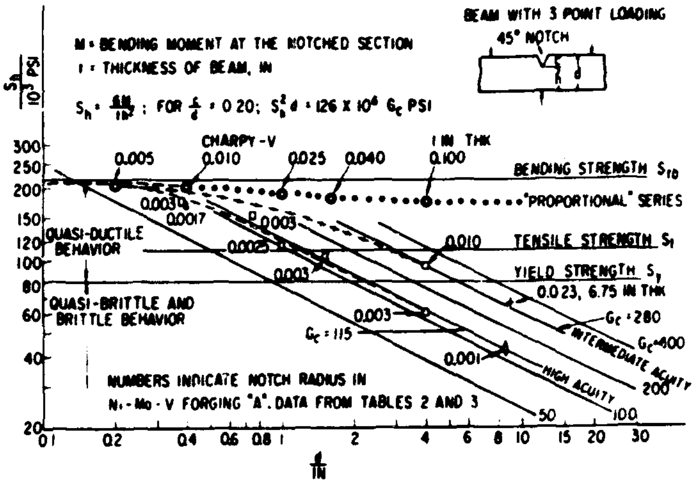

Fig.10 Notch strength versus size. Room temperature slow-bend tests.

0.400, 0.600, 0.900, 1.300 and 2.250 in. The test results are summarized in Table 1 and Fig.9 is a log-log plot of experimental data. The plot is very similar to Fig.5 and the same equation (9) was used to draw the straight lines. The notch strength S_n for small specimens becomes about 155,000 psi which results in a notch strength ratio of 155,000/110,000 = 1.42 quite close to expected ratio 1.50.

It is seen from Fig.9 that the notch-strength-versus-size curve a b c e, drawn through the experimental points is quite similar to one of the family of curves in Fig.7. However, neither point a is definitely located, nor is the point of tangency d between the proposed curve and the straight line which determines the value of wanted fracture toughness G_c. It appears, however, that the six experimental points permit defining the required G_c with sufficient accuracy as 90 lb-in/sq in. The dashed line was drawn using the estimated fracture toughness G_c = 90 lb-in/sq in. and this value is assigned to the foregoing forging in the location where the specimens were removed.

A larger bar from the same forging with D = 3.82 in., with a 50 per cent V-notch, and 0.007 in. notch radius was tested (5, p. 160). This resulted in S_d = 73,000 psi and is included in Fig.9. The larger notch radius causes a higher fracture strength and may be used to justify the choice of 90 in-lb/sq in. for the wanted G_c. The largest specimens with D = 2.25 in. fractured at S_d = 73,500 psi, which is only slightly lower than the yield strength S_y = 82,000 psi. To confirm that the geometrical size effect does take place in accordance with Equation (9) it would have been desirable to test at least one additional bar 5.0 in. in diameter.

With the help of the notch-strength-versus-size curve in Fig.9 it was possible to divide the material behavior into the three regions; namely, ductile, quasi-ductile and quasi-brittle, using the horizontal lines which define the tensile strength and yield strength of unnotched specimens. It is of interest to note that these tests, performed at room temperature, indicate that the ductile behavior extends itself up to 1 in. diam, in spite of the considerably higher, 300 F FATT.

Slow Bend Tests on Notched Specimens

Room-temperature tests of a series of gradually increasing in size, sharply notched, beams are described in (2) and (3). The test material was taken from the same forging A as the notched tension specimens (4). The range of gross beam depth d varied from 3/16 to 8 5/8 in. and the cross section was square. The beams had a 45-deg included angle, 20 per cent depth of notch (c/d) = 0.2, the notch radii varied from 0.001 to 0.100 in. All tests were performed using 3-point loading, and some of the tests are summarized in Tables 2 and 3. In Table 2 are summarized results of tests on beams which were made strictly proportional to a standard Charpy-V impact specimen whose dimensions are: 0.394 in. x 0.394 in., 45 deg V-notch, 0.010 in. radius. The smallest beam was 0.197 in. x 0.197 in. and the largest beam was ten times larger than Charpy-V specimens.

In Table 3 are summarized results of tests on a series of notched beams with all dimensions proportional, except for the notch radii, which were made small, between 0.001 and 0.003 in. The smallest beam was 0.394 in. square and the largest about 9.0 in. square. The nominal bending strength S_n at the notched-beam section is obtained by dividing the maximum bending moment at fracture by the section modulus of the net section.

The test data are plotted in Fig.10 which is a similar representation of notch-strength-versus-size curve for bending as Fig.9 is for notched tension tests. Equation (2) was used to draw the straight lines. The nominal bending strength of a 3/16 in. unnotched beam was determined (3) as 205,000 psi and 227,000 psi, Table 3. The average bending strength, 216,000 psi is approximately twice 110,000 psi, the tensile strength S_t. The foregoing relationship between bending and tension strength is generally recognized and the tests confirmed its applicability to the Ni-Mo-V forging. Tensile strength is not a limiting stress for bending, since for small bars which behave in a ductile manner, when the tensile strength is reached, the bar begins to neck and the process of necking is not sufficiently developed in the deformation accompanying bending of square bars (21, 22). It appears that the foregoing does not apply to the yield strength and it is assumed that the yield strength in bending of an unnotched beam is practically equal to the yield strength in tension of an unnotched specimen.

The series of notched-beam tests with dimensions proportional to Charpy-V bar, summarized in Table 2, are plotted in Fig.10. These tests form a "proportional" series (double circles); at each point the notch radius is given. It can be seen that only a very small and gradual decrease of nominal bending strength with size takes place. In other words, the standard Charpy-V geometry does not lead to appreciable notch-weakening due to geometrical size effect. Also, it does not appear to be possible to assign a reasonable, even if a very large value of fracture toughness, to the curve representing the proportional series. The series of tests with beams with small notch radii, between 0.001 and 0.003 in., but otherwise with proportional dimensions, is summarized in Table 3 and plotted in Fig.10 as the "high-acuity" series. This series of tests indicates considerable notch weakening, a 0.4-in. beam shows a drop of 20 per cent from approximately 210,000 psi and an 8.62-in. square beam with a 0.001-in. radius has a notched beam strength of 43,000 psi, a fifth of the notch-bending strength of a small beam.

TABLE 2*

BEND TESTS OF NOTCHED SQUARE BEAMS AT ROOM TEMPERATURE
ALL DIMENSIONS INCLUDING RADII - PROPORTIONAL
45° V-Notch, 20% Notch Depth

Gross Depth d	Notch Radius r	Nominal Bending Strength S_n	Avg.	Complete Separation at Fracture	Size Factor
in.	in.	psi	psi		
.197	.005	205,000	204,500	No	-
.197	.005	204,000		No	
.394	.010	210,000	204,000	Yes	Charpy
.394	.010	193,000		Yes	
.394	.010	217,000		Yes	Dimensions
.394	.010	195,000		Yes	
1.000	.025	182,000	182,500	Yes	2.5
1.000	.025	183,000		Yes	
1.600	.040	181,000		Yes	4.0
4.000 (1 in. thick)	.100	177,000		Yes	10.0

* From reference (2) Fig. 8 and reference (3) Table I.

TABLE 3*

BEND TESTS OF NOTCHED SQUARE BEAMS AT ROOM TEMPERATURE
BEAMS WITH SHARP NOTCHES
45°, V-Notch, 20% Notch Depth

Gross Depth d	Notch Radius r	Nominal Bending Strength S_n	Avg.	Complete Separation at Fracture
in.	in.	psi	psi	
.394	.0017	169,000	169,000	No
.394	.0017	169,000		No
3/16	.003	198,000	201,000	No
3/16	.003	210,000		No
3/16	.003	194,000		No
3/8	.003	175,000	175,000	No
3/8	.003	175,000		No
3/4	.003	153,000	153,000	Yes
3/4	.003	153,000		Yes
1.0	.0025	116,500		Yes
1 1/2	.003	107,000	104,500	Yes
1 1/2	.003	102,000		Yes
4.0	.003	60,300		Yes
8.62	.001	43,000		Yes
4.0	.010	94,500		Yes
8.94 **	.0225	66,000		Yes
3/16	Unnotched	227,000	216,000	
3/16	"	205,000		

* From reference (2) Fig. 8 and reference (3) Tables I and II
** [illegible]

The last three points in the high-acuity series seem to define a straight line with sufficient accuracy. The nominal notch-bend strength S_n in the case of the 4 and 8.62 in. beams is ll below the yield strength S_y. This indicates at the value of fracture toughness G_c, defined r the straight line as 115 in-lb per sq in., is stified. Tests with "intermediate acuity" consisted of 3 notched beams, 0.197 in., 4.0 in. d 8.94 in., with notch radii 0.005, 0.010 and .0225 in. They are summarized in Tables 2 and and plotted in Fig.10. The notch-strength-versus-size curve indicates again considerable notch akening with size. The notch-bend strength for e last point, 66,000 psi, is lower than the ield strength, and this test point together with e one which corresponds to the 4-in. beam define a straight line with G_c = 280 in-lb per sq 1.

Again it is seen that the experimentally determined notch-strength-versus-size curves justify the concept proposed in Fig.7 that there is a re or less gradual decrease in notch strength om small to large specimens.

The test results of the three series of tched beams from Ni-Mo-V "A" forging indicate at the nominal bending strength of small tched beams of square cross section is slightly lower than the nominal bending strength of untched beams; that is, the notch-strength ratio, r bending of small beams, is slightly less an unity and not considerably more than unity, it was in the case of notched tension specims. Therefore, the foregoing series of tests dicates that although the A steel exhibits, by finition, ductile behavior in a notched tenion test, Fig.9, by the same definition, the eel does not behave in a ductile manner in a tched bend test. However, as in the notched nsion test, the quasi-ductile behavior is evinced also in the bend test. It is confined to e region between the unnotched bending strength d the yield strength. As in the notched tenion test, the quasi-brittle and brittle behavrs extend below the yield strength.

tch Strength of Circular Tension Specimens th Variable Notch Depth

The notch strength of circular tension specens with sharp, 50 per cent notches is plotted Fig.9. It is of interest to determine the riation of notch strength S_d with variation in tch depth N while maintaining constant outer ameter D and fracture toughness G_c. Notch rength S_d is obtainable from Equation (6) with e help of F (N) which is plotted in Fig.4. acture toughness G_c was assumed 90 in-lb per in., the value determined in Fig.9 for materi- A. The notch strength S_d was calculated for ree conveniently chosen outer diameters, D = 1/4 in., 9 in. and 20 1/4 in. It was found advantageous to plot S_d as an ordinate and a function of increasing notch depth N. The results are plotted in Fig.11; curves c, d, and e were calculated for notched bars with outer diameters D = 2 1/4, 9 and 20 1/4, respectively. It is seen that the shape of these curves is defined by the shape of the curve F (N) in Fig.4. For notch depths between 0.35 and 0.65, notch strength S_d is practically constant and has a flat minimum. This minimum is especially noticeable in the case of the two large diameters. Because of flatness, this region of notch depths is most useful for experiments designed to determine G_c.

Equation (6) leads to infinite notch strength for finite G_c and for very shallow notches, $N \doteq 0$, and for very deep notches $N \doteq 1$. Curves c, d, e are drawn in this manner. However, the foregoing is devoid of physical meaning, and actual boundary conditions derived from known physical behavior must be introduced. Obviously, for unnotched specimens, when N = 0, the notch strength S_d becomes either equal to tensile strength S_t in case of small specimens or, in case of much larger diameters, S_d may be somewhat lower. For the large sizes involved it is assumed that the unnotched strength coincides with point p in the middle between S_t and S_y. In order to complete the curves, tangents were drawn from point p to curves e, d and c. For very deep notches, where N is practically 1, it is assumed that all curves pass through point q which coincides with tensile strength S_t. It must be understood that under some conditions point g may be located considerably higher than the tensile strength, closer to points n or m, and this may be especially true when the notch radius is very small. As before, tangents were drawn from point q to complete the curves pcq, pd and peq.

These curves in Fig.11(a) describe the variation of notch-strength of specimens with three different outer diameters made from material A, equipped with sharp notches, for G_c = 90 in-lb per sq in., when their notch depth N is varied from 0 to 1. It follows that the notch strength S_d decreases from the assumed value at p with increasing notch depth, reaches a broad minimum level, and then increases again when the notch becomes much deeper. In case of large diameters, S_d decreases with increasing depth very rapidly, almost abruptly, reaches a broad minimum and again very rapidly increases to reach at least the tensile strength S_t, or possibly a still higher stress. In case of large diameters the minimum which the notch strength S_d assumes may be only a fracture of the yield strength S_y.

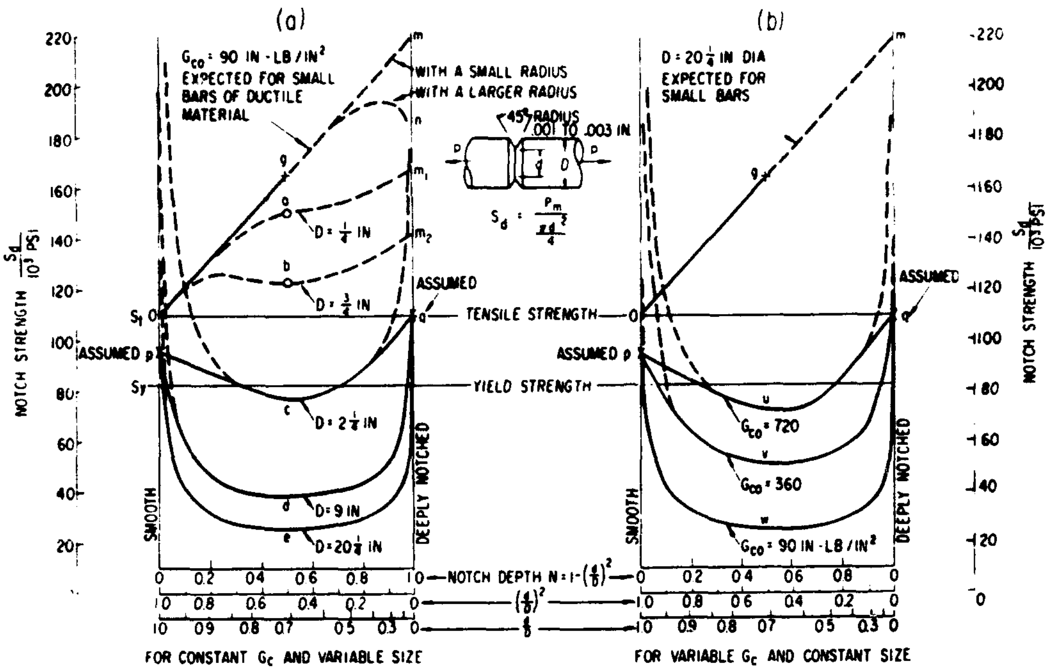

Fig.11 Notch strength versus notch depth curves for cylindrical bars in tension.

or a fixed value of fracture toughness G_c, a articular size (diameter D) may be found for hich the notch strength will be practically contant throughout most of the depth variation. t is seen that shallow and very deep notches re not suitable for the determination of G_c. he stresses in the neighborhood of the notches re limited by the yield and tensile strengths nd therefore, the calculated fracture toughness s too small, (1, p. 1653).

In case of a rather small notched bar, e.g., .1 in. diam, the notch strength S_d may follow he lines ogm or ogn. Fig.9 shows that for D = .25 in., S_d = 150,000 psi and for D = 0.75 in., $_d$ = 124,000 psi. These values define points a nd b, respectively, in Fig.11(a). In the absence of other data, it is assumed that the variation of notch strength for the two specimens ay be represented by curves oam$_1$ and obm$_2$ which ass through the tensile stress S_t and through he two experimentally determined points a and .

A study of the family of curves beginning ith the straight-line ogm to the lowest curve eq, indicates that there is a degree of continuty in respect to the variation of notch strength f specimens with varying notch depth and with ncreasing size. The magnitude of this variaion of notch strength will depend on the sharpess of the notch and on the value of fracture oughness associated with the material.

Fig.11(b) summarizes how notch strength S_d varies when the specimen diameter remains constant, 20 1/4 in., but the fracture toughness assumes three different values G_c = 90,360 and 720 in-lb per sq in. As before, we assume that the notch strength is limited by point p for shallow notches and by point q for very deep notches. Three curves are obtained, puq, pvq, and pwq for the three assumed values of G_c. Curve pwq in Fig.11(b) is identical with curve peq in Fig.11(a). It follows from Fig.11(b) that the regions of notch depth N, where the Griffith-Irwin approach may be used, narrows down considerably from 0.05 to 0.99 to 0.3 to 0.75 when G_c increases from 90 to 720 in-lb per sq in.

The author also has plotted the notch-strength-versus-size curves for a uniformly thick plate in tension, symmetrically notched on the outside. For this, a somewhat modified form of Equation (6) in (1) was used. The same formula was applied to plot notch-strength-versus-notch depth curves, similar to Fig. 11. These will be published at a later date.

A set of curves similar to those in Fig. 11(a) may be obtained for notched beams with the help of Equation (1) and curves A and B in Fig.2.

$$G = \left[(1 - \nu^2)/E\right] \left[1 - (c/d)\right] f(c/d) S_n^2 d \tag{1a}$$

and

$$ {}_{h}{}^{2}d = \left[GE/(1 - \nu^2)\right] \left\{1\Big/\left[1 - (c/d)\right]^2 (c/d)\right\} \tag{1b} $$

e expression

$$ \varphi(c/d) = \left\{1\Big/\left[1 - (c/d)\right]^2 (c/d)\right\}^{1/2} $$

ıs a minimum for (c/d) approximately 0.30. In ;her words, for a constant fracture toughness ; and total beam depth d, the minimum nominal ınding stress S_n will be obtained for a 30 per ınt notch depth. A set of curves for notched ıams, similar to those in Fig.11(a), is preınted in Fig.11 of (2).

It is of interest that, in the case of ›tched disks, the curves representing the net rerage tangential bursting stress as a function f the relative notch depth (1, Fig.11) also ave the same appearance as the curves for the otch strength of cylindrical bars, Figs.11(a) nd 11(b).

dditional Remarks Pertaining to Fracture oughness

The derived expressions for strain-energyelease-rate, G, postulate that the notches were imilar to natural cracks and that linear elasicity applies. Therefore, to obtain fracture oughness experimentally in a manner consistent ith its definition, it is necessary to test a eries of proportional, small and large, speciıens equipped with cracks rather than with mehanically formed notches with finite curvature. racture toughness obtained experimentally with he help of cracks will be designated G_{co}. If he specimens are equipped with very small radii, .g., 0.001 or possibly even 0.003 in., the exerimentally obtained fracture toughness may be ufficiently close to G_{co} to serve, for all pracical purposes, as a substitute. This appears o be the case in Fig.9, G_c = 90 in-lb per sq in. nd in Fig.10, for the high acuity series, G_c = 15 in-lb per sq in. Although both values may e considered as practical substitutes for G_{co}, ne was obtained from tension experiments and he other from bend tests. However, the values ere obtained for the same material A at room emperature, and it appears that this difference f 28 per cent is somewhat large to be accounted or by scatter. Only additional tests with atural cracks may help to dissolve the discrepncy.

Table 7 of (14) summarizes disk bursting ests performed on the same material A. Disk o. 8 was described as sound (no cracks) and it had two radial notches with 0.005 in. radius, extending from the bore. The net average tangential stress of bursting speed was 32,400 psi. With the help of Equation (23) in (1), fracture toughness was calculated as G_c = 107 in-lb per sq in., which falls between 90 in-lb per sq in. obtained for tension and 115 in-lb per sq in., for bending. This rather close agreement is probably coincidental. Although the disk was quite small, only 9 in. diam, 1 in. bore, and 3/8 in. thick, the net average tangential bursting stress was only 40 per cent of the yield strength, 82,500 psi. This indicates that in case of a low value of G_{co}, even a relatively small disk may yield quite satisfactory values of fracture toughness.

The intermediate acuity series in Fig.10 results in a much higher G_c = 280 in-lb per sq in., obviously because of larger notch radii. This difference between the meaning of G_{co} and of G obtained with manufactured notches must be well understood.

It is rather difficult to separate the effect of notch curvature from the effect of size on notch strength. Therefore, it appears logical to define the effect of decreasing notch radius on notch strength by determining experimentally the resulting shift in G_c to lower values, caused by the change in curvature. The proposal to associate the change in G_c with a change in notch radius of a series of specimens is similar to the established procedure to define the degree of temper brittleness by means of a shift in fracture appearance trans. temperature. In the foregoing, it is tacitly assumed that the absolute magnitude of notch radius rather than the ratio of the radius to a characteristic linear dimension defines the effect on the notch strength.

It is obvious from Fig.7 that in order to obtain a dependable value for G_{co}, i.e., to define a straight line, it is advisable to extend the size range far enough to obtain at least two points which are below the yield strength S_y. It is also apparent that for small values of G_{co}, e.g., G_{c1}, any straight line with $tg\alpha = -0.5$ passing through points like b_1, c_1, or d_1 will result in an acceptable lower bound for G_{c1}. However, this is not true for large values of G_{co}, e.g. for G_{c3}. In this case a straight line through point b_3 may not be acceptable, but a straight line through point c_3 may be satisfactory as a lower bound for G_{c3}.

Summary

The formulas and the necessary curves for the calculation of strain-energy release rate for sharply notched beams with 3 and 4-point loading and variable notch depth were summarized in Fig.2. Curves and formulas which permit calculation of strain-energy release rate for a sharply notched cylindrical notched bar in tension were summarized in Figs.3 and 4. It is shown in Fig.5 that a log-log co-ordinate system is especially useful to represent the foregoing relation, because the so-called "geometric size effect," is represented by a straight line with a slope of - 1/2.

It appeared that the change of notch strength was more or less gradual from the higher values for small specimens to substantially smaller values for large specimens. This led to generalization of strength response to sharp notches and resulted in the introduction of the concept of continuous material behavior as represented by the notch-strength-versus-size curves shown in Figs.6 and 7. The somewhat arbitrary classification into ductile, quasi-ductile, quasi-brittle and brittle material behavior on the basis of notch-strength relationship to tensile and yield strength was proposed. It was pointed out that in addition to specimen geometry and crack depth, the recently recognized and important material property "fracture toughness" is necessary in order to be able to establish notch-strength behavior in terms of size. A consequential, but not very obvious, conclusion was inferred, that theoretically, irrespective of the fracture toughness, sharply notched specimens are expected to exhibit quasi-brittle and even brittle behavior, if the size is increased sufficiently, Fig.7.

Two informative, experimentally obtained, room-temperature, notch-strength-versus-size curves, were plotted in Figs.9 and 10 -- for cylindrical notch-tension bars and for notched beams. In Fig.11(a) the notch strength of cylindrical bars was plotted for different outer diameters as a function of notch depth for a constant G_c, and in Fig.11(b) the notch strength is plotted for a constant outer diameter and for variable G_c. The two plots indicate a remarkable continuity of notch strength behavior with varying size and varying notch depth.

As a result of available evidence, it appeared logical to introduce G_{co}, fracture toughness obtainable for a series of test specimens with natural cracks. It was indicated that a shift to lower values of G_c occurs when the notch radii are decreased. It was emphasized that in order to obtain experimentally a satisfactory estimate of G_{co}, notches with the smallest possible radii must be used. It seems that not only the specimen dimension, but also the expected values of G_{co} will influence the choice of radii. It was mentioned that in order to obtain a dependable value for G_{co}, i.e., in order to define a straight line, it is advisable to extend the range of specimens into larger sizes, to obtain at least two points with fracture stresses below the yield strength.

It is hoped that the interpretative analysis of material behavior presented in this paper will contribute to a better understanding of the somewhat cloudy geometric size effect in the presence of notches, and that it will stimulate additional experiments to illuminate some of the less understood phenomena.

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